1212-16-505-001

Code No. 3361 / CORE

FACULTY OF SCIENCE

M.Sc. IV-Semester Examination, May / June 2018

Subject : MATHEMATICS

Paper - I

Advanced Complex Analysis

Time : 3 hours **Max. Marks : 80**

Note : Answer all questions from Part-A and Part-B. Each question carries 4 marks in Part-A and 12 marks in Part-B.

PART – A (8 x 4 = 32 Marks)

1 If $z_1, z_2, \ldots, z_n$ are the zeros of f inside the disc D_R than prove that

$$\int_0^R n(r)\frac{dr}{r} = \sum_{k=1}^{N} \log\left|\frac{R}{z_k}\right|$$

2 Find the growth order of sin πz.

3 Prove that the Gamma function extends to an analytic function in the half plane Re(s) > 0.

4 For $n \in \mathbb{N}$, prove that $\operatorname{Res}_{s=-n} \Gamma(s) = \frac{(-1)^n}{n!}$.

5 Prove that $(\zeta(s))^2 = \sum_{n=1}^{\infty} \frac{d(n)}{n^s}$

6 If Re(s) >l, prove that $\log \zeta(s) = \sum_{p,m} \frac{p^{-ms}}{m}$, where p is prime, $m \in \mathbb{N}$.

7 For $M \in SL_2(\mathbb{R})$, prove that f_M maps H onto itself where H is the upper half plane.

8 Prove that $\psi_\alpha(z) = \frac{\alpha - z}{1-\bar{\alpha}z}$, where $\alpha \in \mathbb{C}$, $|\alpha|<1$ is an automorphism of the unit disc D.

PART – B (4 x 12 = 48 Marks)

9 a) Find the Hadomards products for

i) $e^z - 1$ ii) sin πz

OR

b) State and prove Jensen's formula.

10 a) Prove that $\lim_{n\to\infty} \frac{n^s\, n!}{s(s+1).....(s+n)} = \Gamma(s)$ for $s \neq 0, -1, -2, \ldots.$

OR

b) Prove that $\Gamma(s)\Gamma\left(s+\frac{1}{2}\right) = \sqrt{\pi}\ 2^{1-2s}\,\Gamma(2s)$.

11 a) Prove that, if $\psi_1 \sim \frac{x^2}{x}$ as $x \to \infty$, then prove that $\psi(x) \sim x$ as $x \to \infty$.

OR

b) Show that the function $\xi(s) = \pi^{-\frac{s}{2}} \Gamma\left(\frac{s}{2}\right)\zeta(s)$ is real when s is real or when $Re(s)=\frac{1}{2}$.

12 a) State and prove Schwarz's lemma.

OR

b) Prove that every antomorphism of upper half plane H takes the form f_M for some $M \in SL_2(IR)$.

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FACULTY OF SCIENCE

M.Sc. IV – Semester Examination, May / June 2018

Subject: Mathematics

Paper – II
General Measure Theory

Time: 3 Hours

Max.Marks: 80

Note: Answer all questions from Part-A and Part-B.
Each question carries 4 marks in Part-A and 12 marks in Part-B.

PART – A (8x4 = 32 Marks)
[Short Answer Type]

1 Define a measure μ on a measurable space (x, β). Prove that μ is countably sub additive.

2 State and prove Monotone convergence theorem.

3 Prove that countable union of positive sets ia a positive set.

4 Suppose (x, β, μ) is a measure space and f is an integrable function on X w.r.t. μ. Prove that ν defined on β by $\nu(E) = \int_E f d\mu$ is a signed measure on β.

5 Define a μ^*-measurable set E. Suppose. $\mu^*(E) = 0$, prove that E is a μ^*-measurable set.

6 Suppose $E \subset x$ xy and $x \in x$. Define x – cross section of E with usual notations. Prove that

i) $\psi_{E_x}(y) = \psi_E(x, y) \ \forall\, y \in y$

ii) $\bar{E}_x = (\bar{E})_x$

7 Suppose μ^* and μ_* are the outer and inner measures induced by a measure μ on an algebra A of subsets of X. Prove that $\mu_*(E) \le \mu^*(E) \ \forall\, E \in P(x)$.

8 If A ∊ A prove that

$\mu(A) = \mu_*(A \cap E) + \mu^*(A \cap \bar{E})$.

PART – B (4x12 = 48 Marks)
[Essay Answer Type]

9 a) Suppose(x, β, μ) is a measure space. Prove that is can be extended to a complete measure space (x_1, β_0, μ_0) where $\beta \subset \beta_0$ and restriction μ_0 to β is μ i.e. $\mu_0 / \beta = \mu$.

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