Code No: 7092/CORE

FACULTY OF SCIENCE

M.Sc. I Semester Examinations, December 2018 / January 2019

Subject: Maths/Applied Maths/Mathematics with Computer Science

Paper: I - Abstract Algebra

Time: 3 Hours **Max. Marks: 80**

Note: Answer all questions from Part A and Part B. Each question carries 4 marks in Part – A and 12 marks in Part – B.

PART – A (8x4=32 Marks)
(Short Answer Type)

1. Give an example of a group G such that G has a normal subgroup H with both H and $\frac{G}{H}$ Nilpotent but G not nilpotent.
2. Prove that any group of order P^n, P a prime, is nilpotent.
3. Show that the group $\left(\frac{Z}{(8)},+\right)$ cannot be written as the direct sum of two non-trivial subgroups.
4. Prove that there are only two non-abelian group of order 8.
5. Let R be a commutative ring with unity. Suppose R has no non-trivial ideas. Prove that R is a field.
6. Prove that $\frac{Z[x]}{\langle x^2+1\rangle} \simeq Z[i], where\ Z[i]=\{a+b\sqrt{-1}: a,b\in z\}$ is the ring of Gaussian integers.
7. Show that 3 is irreducible but not prime in the ring $Z\left[\sqrt{-5}\right]$
8. Prove that Z[i] is a Euclidean Domain.

PART – B (4x12=48 Marks)
(Essay Answer Type)

9. (a) Let G be a nipolent group. Then prove that every subgroup of G and every homomorphic image of G is nilpotent.

OR

(b) Derive class equation of a finite group G.

10. (a) Let (A, +) be a finitely generated abelian group. Then prove that A can be decomposed as a direct sum of a finite number of cyclic groups.

OR

(b) (i) State and prove Cauchy's Theorem for finite abelian groups.
(ii) Prove First sylow theorem.

11. (a) Suppose R is a non-zero ring with unity. Let $I(\neq R)$ be an ideal in R. Prove that there is a maximal ideas M in R such that $I\subseteq M$

OR

(b) Suppose R is a commutative ring and P is an ideal in R. Then prove that P is a prime ideal in R if and only if "$a,b\in P,\ a\in R, b\in R \Rightarrow a\in p\ or\ b\in p$".

12. (a) If R is a UFD, then prove that R[x] is also a UFD over R.

OR

(b) (i) Prove Gauss lemma.
(ii) Prove division algorithm in polynomial rings.

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FACULTY OF SCIENCE

M.Sc. I Semester Examinations, December 2018 / January 2019

Subject: Maths/Applied Maths/ Mathematics with Computer Science

Paper- II : Mathematical Analysis

Time: 3 Hours **Max. Marks: 80**

Note: Answer all questions from Part A and Part B. Each question carries 4 marks in Part – A and 12 marks in Part – B.

PART – A (8x4=32 Marks)
(Short Answer Type)

1. Prove that a set E is open if and only if its complement is closed.
2. If $\{I_n\}$ is a sequence of intervals in R^1, such that $I_n \supset I_{n+1}$ $(n=1,2,3....)$, then $\bigcap_{m=1}^{\infty} I_n$ is not empty.
3. Prove that continuous image of a compact metric space is compact.
4. Let f be a monotonically increasing function defined on (a,b). Then prove that the set E of all discontinuities of f is at most countable.
5. Prove that $\int_a^b f d\alpha \le \int_a^{\overline{b}} f d\alpha$
6. Define Rectifiable curve.
7. State and prove M_n-test.
8. Show that the series $\sum_{n=0}^{\infty} \frac{1}{n^p + n^q x^2}$ is uniformly convergent for all values of x if p>1

PART – B (4x12=48 Marks)
(Essay Answer Type)

9. (a) Prove that every k-cell is compact.

OR

(b) Prove that every nonempty perfect set in R^k is uncountable.

10. (a) Prove that a mapping f of a metric space X into a metric space Y is continuous on X if only $f^1(V)$ is open in X for every open set V in Y.

OR

(b) Let E be a non-compact sub set of R, then prove that

(i) there exists a continuous function on E which is not bounded.

(ii) there exists a continuous and bounded function on E which does not attains its supremum.

11. (a) State and prove a necessary and sufficient condition for a bounded function f to be Riemann Stieltjes integrable on [a,b].

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