PART – A (5 x 4 = 20 Marks)  
(Short Answer Type) 
Note: Answer any FIVE of the following questions.

1. Expand \( f(x) = \log (1 + \sin x) \) by using Maclaurin’s theorem.

2. Find the value of \( c \) in Rolle’s mean value theorem for the function 
   \[ f(x) = \log \left( \frac{x^2 + ab}{x(a + b)} \right) \text{ on } [a, b]. \]

3. Evaluate \( \lim_{x \to 0} (\cot x)^{\tan x} \).

4. Find the radius of curvature of the curve \( x = a \cos^2 t, \ y = b \sin^2 t \) at \( t = \pi/4 \).

5. If \( z = \log (u^2 + v), \ u = e^{xy}, \ v = x + y^2 \) then evaluate \( 2y \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} \).

6. If \( u = \tan \left( \frac{y^2}{x} \right) \) then evaluate 
   \[ x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}. \]

7. Find the asymptotes of the curve \( x^2 y^2 - x^2 y - xy^2 + x + y + 1 = 0 \), which are parallel to coordinate axes.

8. Find the envelope of the curve \( my + m^2 x - 10 = 0 \) where \( m \) is a parameter.

PART – B (4 x 15 = 60 Marks)  
(Essay Answer Type) 
Note: Answer ALL the questions.

9. (a) (i) \( y = a \cos (\log x) + b \sin (\log x) \) then show that
   \[ x^2 y_{n+2} + (2n + 1) x y_{n+1} + (n^2 + 1) y_n = 0 \]
   (ii) Find the coefficient of \( x^3 \) in Maclaurin’s series expansion of \( f(x) = e^x \cos x \).

   OR

   (b) (i) State and prove Cauchy’s mean value theorem. Hence find ‘c’ value of
   Cauchy’s mean value theorem for the function \( f(x) = e^x, \ g(x) = e^{-x} \) on \([a, b]\).
10 (a) (i) Find the curvature, the radius of curvature and the centre of the circle of curvature and the circle of curvature for the curve \(x^2 = 4ay\) at \(P(2a, a)\).
(ii) Find the evolute of the parabola \(y^2 = 4ax\).

OR

(b) (i) Evaluate \(\lim_{x \to 0} \frac{(1 + x)^{\frac{1}{x}} - e + \frac{ex}{2}}{x^2}\).
(ii) Find the value of \(a\) and \(b\) so that \(\lim_{x \to 0} \frac{x(1 + a \cos x) - b \sin x}{x^2} = 1\)

11 (a) (i) If \(f(x, y) = \begin{cases} \frac{xy(x^2 - y^2)}{x^2 + y^2}, & (x, y) \neq (0,0) \\ 0, & (x, y) = (0,0) \end{cases}\) then show that \(f_x(0,0) \neq f_y(0,0)\).
(ii) If \(u = \log(x^3 + y^3 + z^3 - 3xyz)\) then show that \(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x + y + z}\)

OR

(b) (i) State and prove Euler's theorem for a homogeneous function.
(ii) Explain \(f(x, y) = x^2y + 3y - 2\) in terms \(x + 1, y - 2\) as a Taylor series.

12 (a) (i) Find the minimum value of \(x^2 + y^2 + z^2\) subject to the condition \(xyz = a^3\) where \(a > 0\).
(ii) Discuss the maximum and minimum values of \(f(x, y) = xy + \frac{9}{x} + \frac{3}{y}\)

OR

(b) Find the asymptotes of the curve.
\[x^3 + 2x^2y - xy^2 - 2y^3 + xy - y^2 - 1 = 0\]

****