FACULTY OF SCIENCE
B.Sc. I-Semester (CBCS) Examination, December 2017
Subject: Mathematics
Paper – I
Differential Calculus

Time: 3 Hours
Max. Marks: 80

PART– A (5x4 = 20 Marks)
[Short Answer Type]

Note: Answer any FIVE of the following questions.

1. Find the $n^{th}$ derivative of $f(x) = \frac{1}{6x^2 - 5x + 1}$

2. Expand $f(x) = e^x$ in powers of $(x-2)$.

3. Evaluate \( \lim_{x \to 1} \frac{x \cos x - \log (1+x)}{x^2} \)

4. Find the radius of curvature of the curve $x^4 + y^4 = 2$ at the point $P(1,1)$.

5. If $w = x^2 + y^2$, $x = r\cdot s$ and $y = r+s$ then evaluate $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$.

6. If $z = f(x+ay) + g(x-ay)$ then show that $\frac{\partial^2 z}{\partial y^2} = a^2 \frac{\partial^2 z}{\partial x^2}$.

7. Find the envelope of the family of circles $x^2 + y^2 - 2ax \cos \alpha - 2ay \sin \alpha = c^2$ where $\alpha$ is the parameter.

8. Find the asymptotes of the curve $\gamma = \frac{a\theta}{\theta - 1}$.

PART– B (4x15 = 60 Marks)
[Essay Answer Type]

Note: Answer ALL the questions.

9. a) If $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$ then show that $x^2 y_{n+2} + (2n+1) xy_{n+1} + 2n^2 y_n = 0$.

OR

b) i) State and prove Rolle's mean value theorem.

ii) If Rolle's mean value theorem holds for the function $f(x) = x^3 + ax^2 + bx$, $1 \leq x \leq 2$ at the point $x = \frac{4}{3}$ then find the values of $a$ and $b$. 
10 a) Find the circle of curvature of the curve \( x = a(\cos t + t \sin t), \ y = a(\sin t - t \cos t) \) at 
\( t = \frac{\pi}{4} \).

b) i) Evaluate \( \lim_{x \to 0} \left( \frac{1}{x^2} - \frac{1}{\sin^2 x} \right) \).

ii) Evaluate \( \lim_{x \to \pi} (\tan x)^\tan 2x \).

11 a) i) If \( u = \cos^{-1} \left( \frac{x + y}{\sqrt{x} + \sqrt{y}} \right) \) then show that 
\( x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + \frac{1}{2} \cot u = 0 \).

ii) If \( x^y + y^x = a^b \) then find \( \frac{dy}{dx} \).

b) Expand \( f(x,y) = e^x \cos y \) in terms of \( x-1 \) and \( \left( y - \frac{\pi}{4} \right) \) using Taylor's theorem.

12 a) Find the asymptotes of the curve \( x^3 - 6x^2 y + 11x^2 y - 6y^3 + x + y + 1 = 0 \).

b) Find the minimum value of \( x + y + z \), subject to the condition \( \frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 1 \).