1212-14-468-023

Code No. 7006 / E

FACULTY OF SCIENCE

B.Sc. I-Semester (CBCS) Examination, December 2017

Subject: Mathematics

Paper – I
Differential Calculus

Time: 3 Hours **Max.Marks: 80**

PART – A (5x4 = 20 Marks)
[Short Answer Type]

Note: Answer any FIVE of the following questions.

1 Find the n^{th} derivative of $f(x) = \dfrac{1}{6x^2 - 5x + 1}$.

2 Expand $f(x) = e^x$ in powers of $(x-2)$.

3 Evaluate $\lim\limits_{x \to 0} \dfrac{x \cos x - \log(1+x)}{x^2}$

4 Find the radius of curvature of the curve $x^4 + y^4 = 2$ at the point P(1,1).

5 If $w = x^2 + y^2$, $x = r-s$ and $y = r+s$ then evaluate $\dfrac{\partial w}{\partial r}$ and $\dfrac{\partial w}{\partial s}$.

6 If $z = f(x+ay) + g(x-ay)$ then show that $\dfrac{\partial^2 z}{\partial y^2} = a^2 \dfrac{\partial^2 z}{\partial x^2}$.

7 Find the envelope of the family of circles $x^2+y^2 - 2ax\cos\alpha - 2ay\sin\alpha = c^2$ where α is the parameter.

8 Find the asymptotes of the curve $\gamma = \dfrac{a\theta}{\theta - 1}$.

PART – B (4x15 = 60 Marks)
[Essay Answer Type]

Note: Answer ALL the questions.

9 a) If $\cos^{-1}\left(\dfrac{y}{b}\right) = \log\left(\dfrac{x}{n}\right)^n$ then show that $x^2 y_{n+2} + (2n+1)xy_{n+1} + 2n^2 y_n = 0$.

OR

b) i) State and prove Rolle's mean value theorem.

ii) If Rolle's mean value theorem holds for the function $f(x) = x^3 + ax^2 + bx$, $1 \le x \le 2$ at the point $x = \dfrac{4}{3}$ then find the values of a and b.

10 a) Find the circle of curvature of the curve $x = a(\cos t + t \sin t)$, $y = a(\sin t - t \cos t)$ at $t = \frac{\pi}{4}$.

OR

b) i) Evaluate $\lim_{x \to 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x} \right)$.

ii) Evaluate $\lim_{x \to \frac{\pi}{4}} (\tan x)^{\tan 2x}$.

11 a) i) If $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}} \right)$ then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}\cot u = 0$.

ii) If $x^y + y^x = a^b$ then find $\left(\frac{dy}{dx} \right)$.

OR

b) Expand $f(x,y) = e^x \cos y$ in terms of $(x-1)$ and $\left(y - \frac{\pi}{4} \right)$ using Taylor's theorem.

12 a) Find the asymptotes of the curve $x^3 - 6x^2y + 11xy^2 - 6y^3 + x + y + 1 = 0$.

OR

b) Find the minimum value of $x + y + z$, subject to the condition $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 1$.

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