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Code No: 7114/E/R

FACULTY OF SCIENCE

B.Sc. IV – Semester (CBSC) Examination, June 2018
Subject: Mathematics
Paper: IV Algebra

Time: 3 Hours **Max. Marks: 80**

SECTION – A (5 x 4 = 20 Marks)
(Short Answer Type)
Note: Answer any Five of the following questions

1. Write all subgroups of the group $\mathbf{Z}_{30}$ and indicate their orders.
2. For n>1, show that the alternating group A_n has order $\frac{n!}{2}$
3. If G is a group and H is a sub group of index 2 in G. then show that H is a normal subgroup of G.
4. If G is an abelian group and H is a normal subgroup of G then show that $\frac{G}{H}$ is also an abelian group.
5. Define idempotent element in a ring R. Find all idempotent elements in the ring $(Z_{10}, +_{10}, \times_{10})$
6. If I_1 and I_2 are any two ideals in a ring R, then show that $I_1 \cap I_2$ is always an ideal of R.
7. If $f(x) = 1+2x+3x^2$, $g(x) = 2+3x+4x^2+x^3$ then evaluate f(x)+g(x), f(x).g(x) in the ring $Z_5[x]$.
8. Let R be a commutative ring of characteristic 2 then show that the mapping $\phi : R \to R$ Defined by $\phi(a) = a^2 \ \forall a \in R$ is a homomorphism.

SECTION-B (4x15=60 Marks)
(Essay Answer Type)

9. (a) (i) Let G be a group and H, K be two subgroups of G. Then show that HK= $\{hk | h \in H, K \in K\}$ is a subgroup of G.

(ii) Let G be a group and $a \in G$ is such that o(a) = n then show that $o(a^k) = \frac{n}{\gcd(n,k)}$ (where k is a positive integer)

OR

(b) (i) If $\alpha = (a_1, a_2, a_3 \ldots\ldots a_m)$ and $\beta = (b_1, b_2, b_3 \ldots\ldots b_n)$ are any two disjoint permutations then show that $\alpha\beta = \beta\alpha$

(ii) Let $\alpha, \beta \in S_6$ and $\alpha = (1\,2\,4\,5\,3\,6), \beta = (1\,4\,3\,2\,5\,6)$ then evaluate $\alpha.\beta, \alpha\beta^{-1}, \alpha^2$.

10. (a) Let G be a group and a,b $\in$ G and H is a subgroup of G then show that
(i) $aH = bH \Leftrightarrow a \in bH$
(ii) ah is a sub group of G $\Leftrightarrow a \in H$.

OR

(b) Let G be a finite abelian group and P be a prime that divides the order of G then show that G has an element of order P.

11. (a) (i) Show that every finite integral domain is a field.
(ii) Define characteristics of a ring R with unity. Show that the characteristics of an integral domain is either zero or a prime.

OR

Contd..2..

(b) (i) Let R be a commutative ring with unity and A be an ideal of R then show that the quotient ring $\frac{R}{A}$ is an integral domain if and only if A is a Prime ideal.

(ii) Let I be an ideal of a ring R, $1 \in I$ then show that I = R.

12. (a) (i) Define kernel of a ring homomorphism.

(ii) Let R be a ring and A is an ideal of R. Then show that the mapping $\phi : R \to \frac{R}{A}$ defined $\phi(x) = x+A \ \forall \ x \in R$ is an onto homomorphism.

OR

(b) Let F be a field and f(x), g(x) $\in$ F[x] with g(x) $\neq 0$ then show that there exists unique polynomials q(x) and r(x) such that f(x) = q(x) + r(x) with either r(x)= 0 or deg r(x)<deg g(x)

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