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Code No. 8042

FACULTY OF SCIENCE

M. Sc. I – Semester Examination, December 2015

Subject : Maths / Applied Maths / Maths with Computer Science

Paper – I : Algebra

Time : 3 Hours **Max. Marks: 80**

Note : Answer all questions from Part–A and Part–B. Each question carries 4 marks in Part–A and 12 marks in Part – B.

PART – A (8 x 4 = 32 Marks)
(Short Answer Type)

1 Let G be a group, H < G. Then prove that the set $\frac{G}{H}$ of cosets is a G-set under an appropriate action of G on $\frac{G}{H}$.

2 If G is a group and $|G|=p^n$, n > 0 and p a prime then prove that Z(G) is non-trival.

3 Show that the group $\left(\frac{Z}{(8)},+\right)$ can't be written as the direct sum of two nontrivial subgroups.

4 Let G be a group of order pq, where p and q are primes such that p > q and $q \nmid p-1$. Then prove that G is a cyclic group.

5 Prove that the ring of Gaussian integers is a Euclidean domain.

6 Prove that an ideal M in the ring Z of integers is a maximal ideal if and only if M = (p) for some prime p.

7 Let R be a commutative ring and p a prime ideal. Then prove that S = R – P is a multiplicative set.

8 If M is an R-module and $x \in M$ then prove that the set $Rx = \{rx \mid r \in R\}$ is an R-submodule of M.

PART – B (4 x 12 = 48 Marks)
(Essay Answer Type)

9 (a) (i) Prove that every group of order p^2 (p prime) is abelian.

(ii) Prove that a group g is solvable if and only if it has a normal series with abelian factors.

OR

(b) (i) State and prove Jordon Holder theorem.

(ii) Let G be a finite group, $N \Delta G$ and $(|N|, |\frac{G}{N}|) = 1$. Show that every element of order dividing |N| is contained in N.

10 (a) State and prove the fundamental theorem on finitely generated abelian groups.

OR

(b) State and prove First Sylow theorem.

11 (a) If R is a UFD prove that R[x] is also a UFD.

OR

(b) For any ring R and any ideal $A \neq R$, prove that the following statements are equivalent:

(i) A is maximal

(ii) $\frac{R}{A}$ has no non trivial ideals

(iii) $x \in R - A \Rightarrow A + (x) = R$

12 (a) Let $\{N_i\}_{i \in \Delta}$ be a family of R-submodules of an R-module M. Then prove that the following are equivalent.

(i) $\sum_{i \in \Delta} N_i$ is a direct sum

(ii) $0 = \sum_i x_i \in \sum_i N_i \Rightarrow x_i = 0$ for all i

(iii) $Ni \cap \sum_{\substack{j \in \Delta \\ j \neq i}} N_j = 0, \quad i \in \Delta$

OR

(b) (i) State and prove fundamental theorem of R-homomorphisms

(ii) Let A and B be R-submodules of R-modules M and N respectively. Then prove that

$$\frac{M \times N}{A \times B} \simeq \frac{M}{A} \times \frac{N}{B}$$

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