Code No. 8043

FACULTY OF SCIENCE

M. Sc. I – Semester Examination, December 2015

Subject : Maths / Applied Maths / Maths with Computer Science

Paper – II : Real Analysis

Time : 3 Hours **Max. Marks: 80**

Note : Answer all questions from Part–A and Part–B. Each question carries 4 marks in Part–A and 12 marks in Part – B.

PART – A (8 x 4 = 32 Marks)
(Short Answer Type)

1 If Σa_n converges absolutely then prove that Σa_n converges and also show that converse need not be true by an example.

2 If f is a continuous mapping of a compact metric space X into a metrics space Y then prove that f(x) is compact.

3 If p* is a refinement of partition p then prove that
$U(p^*, f, \alpha) \leq U(p, f, \alpha)$.

4 If $f \in R(\alpha)$ and $g \in R(\alpha)$ a [a, b] then prove that $fg \in R(\alpha)$.

5 If $\lim_{n\to\infty} f_n(x) = f(x)$ and if $M_n = \sup_{x\in E} | f_n(x) - f(x)|$ then prove that f_h converge to f uniformly on E if and only if M_n converge to zero.

6 If $f_n(x)=n^2x(1-x^2)^n$, $0\leq x \leq 1$, n = 1, 2, ……., then show that
$$\lim_{n\to\infty} \int_0^1 f_n(x)dx \neq \int_0^1 [\lim_{n\to\infty} f_n(x)]dx.$$

7 If a vector space X is spanned by a set of r vectors then show that $\dim X \leq r$.

8 Prove that a linear operator A on a finite dimensional vector space X is one to one if and only if the range of A is all of X.

PART – B (4 x 12 = 48 Marks)
(Essay Answer Type)

9 (a) If f is a continuous mapping of a compact metric space X into metric space Y then prove that f is uniform continuous on X.

OR

(b) If f is a continuous mapping of metric space X into a metric space Y and if E is connected subset of X then prove that f(E) also connected also prove that if $f(a) < f(b)$ on the interval [a, b] and if $f(a) < c < f(b)$ then $\exists$ a point $x \in (a, b) \ni f(x) = C$.

10 (a) If α increases monotonically and $\alpha' \in R$ on [a, b] and if f is bounded on [a, b] then prove that $f \in R(\alpha)$ if an only if $f\alpha' \in R$ and also show that $\int_a^b f d\alpha = \int_a^b f(x)\alpha'(x)dx$.

OR

(b) If $f_1, f_2 \in R(\alpha)$ on [a, b] and c is any constant then prove that $f_1+f_2 \in R(\alpha)$ and $cf_1 \in R(\alpha))$ and also show that $\int_a^b (f_1+f_2)d\alpha = \int_a^b f_1 d\alpha + \int_a^b f_2 d\alpha$ and $\int_a^b c\ f d\alpha = c\int_a^b f_1 d\alpha$

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11 (a) If $\{f_n\}$is a sequence of function differentiable on [a, b] and if $[f_n(x_o)]$ converge for some point x_o and [a, b] and also if $[f'_n]$ converge uniform on [a, b] then prove that $[f_n]$ converge uniformly on [a, b] to a function f and also show that $f^1(x) = \lim_{n\to\infty} f_n^1(x)$, $a \leq x \leq b$.

OR

(b) State and prove Weicrstrass approximation theorem.

12 (a) If E is an open set in R^n, f maps E into R^m, f is differentiable at $x_o \in E$, g maps an open set contains f(E) into R^k and g is differentiable at $f(x_o)$ then prove that the mapping F of E into R^k defined by F(x) = g (f(x) is differentiable at x_o and $F'(x_o)=g'(f(x_o)) f'(x_o)$.

OR

(b) Define contraction mapping and also state and prove contraction principle.

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