

## FACULTY OF SCIENCE

M. Sc. I – Semester Examination, December 2015

Subject : Mathematics

Paper – IV : Elementary Number Theory

Time : 3 Hours

Max. Marks: 80

Note : Answer all questions from Part–A and Part–B. Each question carries 4 marks in Part–A and 12 marks in Part – B.

PART – A (8 x 4 = 32 Marks)

(Short Answer Type)

- 1 Prove that every integer  $n > 1$  is either a prime number or a product of prime numbers
- 2 If  $d|n$  and  $d|m$  then prove that  $d|an + bm$  for some integers  $a$  and  $b$ .
- 3 For  $n \geq 1$  prove that  $\sum_{d|n} \mu(d) = \left[ \frac{1}{n} \right]$ .
- 4 State and prove Mobius inversion formula.
- 5 If  $(k, m) = 1$  and  $\{a_1, a_2, \dots, a_m\}$  is a complete residue system modulo  $m$  then prove that  $\{ka_1, ka_2, \dots, ka_m\}$  is also complete residue system mod  $m$ .
- 6 State and prove Euler-Fermat theorem.
- 7 Prove that Legendre's symbol  $\left( \frac{n}{p} \right)$  is a completely multiplicative function of  $n$ .
- 8 For every odd prime  $p$  prove that  $\left( \frac{2}{p} \right) = (-1)^{\frac{p^2-1}{8}}$ .

PART – B (4 x 12 = 48 Marks)

(Essay Answer Type)

- 9 (a) State and prove fundamental theorem of arithmetic.  
OR  
(b) State and prove the division algorithm.
- 10 (a) (i) For  $n \geq 1$  prove that  $\phi(n) = \sum_{d|n} \mu(d) \frac{n}{d}$ .  
(ii) If  $f$  and  $g$  are multiplicative prove that their Dirichlet product  $f * g$  is also multiplicative.  
OR  
(b) (i) Let  $f$  be a multiplicative function. Prove that  $f$  is completely multiplicative if and only if  $f^{-1}(n) = \mu(n)f(n)$  for all  $n \geq 1$ .  
(ii) Show that  $\mu$  is multiplicative.
- 11 (a) (i) State and prove Lagrange's theorem.  
(ii) If  $(a, m) = d$  then prove that the linear congruence  $ax \equiv b \pmod{m}$  has solution and only if  $d|b$ .  
OR  
(b) (i) State and prove Chinese remainder theorem.  
(ii) Solve  $x \equiv 2 \pmod{3}$ ,  $x \equiv 3 \pmod{5}$ ,  $x \equiv 2 \pmod{7}$ .
- 12 (a) State and prove Euler's criterion.  
OR  
(b) State and prove Gauss lemma.

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