FACULTY OF SCIENCE
M. Sc. I – Semester Examination, December 2015
Subject: Maths / Applied Maths
Paper – V: Mathematical Methods

Time: 3 Hours

Max. Marks: 72

Note: Answer all questions from Part-A and Part-B. Each question carries 4 marks in Part-A and 12 marks in Part-B.

PART – A (8 x 4 = 32 Marks)
(Short Answer Type)

1. Apply Picard’s method to solve initial value problem up to third approximation
   \( y' = 2y - 2x^2 - 3 \), given that \( y = 2 \) when \( x = 0 \).
2. Solve \( y'p = 2xy + \log q \).
3. Solve \( x^2 \) where \( p, r \), are first and second partial derivatives with respect to \( x \).
4. Classify the equation \( \frac{\partial^2 u}{\partial x^2} + 4 \frac{\partial^3 u}{\partial x \partial y} + 4 \frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial x} + 2 \frac{\partial u}{\partial y} = 0 \).
5. Solve \( \frac{d^2 y}{dx^2} - y = x \), using power series method.
6. Solve \( \frac{\partial u}{\partial x} = \frac{\partial u}{\partial y} \) with \( u(x, 0) = 4e^x \) using separation of variable method.
7. More that \((n+1)L_{n+1}(x) = (2n+1-x)L_n(x) - nL_{n-1}(x)\).
8. Show that \( L_{2n+1}(0) = 0 \).

PART – B (4 x 12 = 48 Marks)
(Essay Answer Type)

9. (a) Prove that all eigenvalues of Strum-Liouville’s problem are real.
   OR
   (b) Explain Charpit’s method for \( p, q \) from \( f(x, y, z, p, q) = 0 \) and hence find the solution to \( 2xz - px^2 - 2qxy + pq = 0 \).

10. (a) Reduce \( \frac{\partial^2 z}{\partial x^2} = x \cdot \frac{\partial^2 z}{\partial y^2} \) to a canonical form and hence solve it.
   OR
   (b) Solve \( \frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2} \) subject to the following condition
       (i) \( u(0, t) = 0 \)  (ii) \( \frac{\partial u}{\partial x} = 0 \) for \( x = \ell \)  (iii) \( u(x, 0) = u_0 \frac{x}{\ell} \) for \( 0 \leq x \leq \ell \).

11. (a) Solve by Frobenius method
    \( x^2 \frac{d^2 y}{dx^2} + 2x \frac{dy}{dx} + 2y = 0 \)
    OR
    (b) State and prove orthogonal properties of Legendre’s polynomials.

12. (a) State and prove orthogonal properties of Hermite polynomials.
    OR
    (b) Prove that \( \int_0^\infty e^{-x}L_m(x)L_n(x)dx = 0 \); if \( m \neq n \)
    \( = 1 \); if \( m = n \)

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