FACULTY OF SCIENCE
Subject: Mathematics/Applied Mathematics / Maths with Computer Science

Paper – I: Algebra

Time: 3 Hours Max. Marks: 80

Note: Answer all questions from Part–A and Part–B. Each question carries 4 marks in Part–A and 12 marks in Part – B.

PART – A (8 x 4 = 32 Marks)
(Short Answer Type)

1. Prove that every finite group has a composition series.
2. Show that every homomorphic image of a solvable group is solvable.
3. Prove that a finite group G is a P-group if its order is power of P.
4. Show that a group of order 1986 is not simple.
5. Suppose f : F → R is a non-zero homomorphism of a field F into a ring R. Then prove that f is one-one.
6. Show that if A and B are nilpotent ideals, their sum A + B is nilpotent.
7. Prove that an irreducible element in a commutative principle ideal domain is always prime.
8. If f(x), g(x) ∈ R(x) then prove that C(fg) = C(f)C(g), where C(f) denotes the content of f(x).

PART – B (4 x 12 = 48 Marks)
(Essay Answer Type)

9. (a) Derive class equation of a finite group.

OR

(b) Prove that a group G is solvable if and only if G has a normal series with abelian factors.

10. (a) (i) State and prove Cauchy's theorem for finite abelian groups.

(ii) Prove that there is a 1–1 correspondence between the family F of non-isomorphic abelian groups of order P^n. P prime, and the set P(e) of partitions of e.

OR

(b) (i) If the order of a finite group G is divisible by a prime power P^n, then prove that G has subgroup of order P^n.

(ii) Prove that a a sylow P-subgroup of a finite group G is unique if and only if it is normal.

11. (a) State and prove correspondence theorem for rings.

OR

(b) If R is a non-zero ring with unity and I is an ideal in R such that I ≠ R then prove that there exists a maximal ideal M of R such that I ⊆ M.

12. (a) Prove that every PID is a UFD but a UFD need not be a PID.

OR

(b) If R is a unique factorization domain, then prove that the polynomial ring R[x] over R is also a unique factorization domain.

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