1212-16-505-001

Code No. 6067/CBCS

FACULTY OF SCIENCE

M. Sc. I – Semester (CBCS) Examination, December 2016

Subject : Maths / Applied Maths / Maths with Computer Science

Paper – II : Analysis

Time : 3 Hours **Max. Marks: 80**

Note : Answer all questions from Part–A and Part–B. Each question carries 4 marks in Part–A and 12 marks in Part – B.

PART – A (8 x 4 = 32 Marks)
(Short Answer Type)

1 Prove that the closed subsets of compact sets are compact.

2 Define a metric space and the cantor set.

3 If f and g are complex continuous functions on a metric space X. Then prove that f + g is continuous on X.

4 Prove that monotonic functions have no discontinuities of the second kind.

5 Prove that $\underline{\int_a^b} f\,d\alpha \le \overline{\int_a^b} f\,d\alpha$.

6 If f is continuous on [a, b] then prove that $f \in R(\alpha)$ on [a,b].

7 State and prove Cauchy criterion for uniform convergence of a sequence of functions.

8 Examine the uniform $\sum_{n=1}^{\infty}(-1)^n \frac{x^2+n}{n^2}$ convergence of on any bounded interval.

PART – B (4 x 12 = 48 Marks)
(Essay Answer Type)

9 (a) (i) Prove that every K – cell is compact. (8)
(ii) Prove that every neighborhood is an open set. (4)

OR

(b) (i) If P be a nonempty perfect set in R^k. Then prove that P is uncountable.
(ii) Is $d(x, y) = (x- y)^2$ for $x, y \in R$ metric or not? Justify your answer.

10 (a) Let f be a continuous mapping of a compact metric space X into a metric space Y. Prove that f is uniformly continuous on X.

OR

(b) Let f be a monotonically increasing function on (a, b) and $x \in (a, b)$. Prove that the following.
(i) $\sup_{a<t<x} f(t) = f(x^-) \le f(x) \le f(x^+) = \inf_{x<t<b} f(t)$ and
(ii) If $a < x < y < b$, then $f(x^+) \le f(y^-)$

11 (a) Suppose $f \in R(\alpha)$ on [a, b], $m \le f \le M$, ϕ is continuous on [m, M], and $h(x) = \phi(f(x)$ on [a, b]. Then prove that $h \in R(\alpha)$ on [a, b].

OR

(b) Assume α increases monotonically and $\alpha' \in R$ on [a, b]. Let f be a bounded function on [a, b]. Then prove that $f \in R(\alpha) \Leftrightarrow f\alpha' \in R$. More over $\int_a^b f\, d\alpha = \int_a^b f\alpha' dx$ in the case $f \in R(\alpha)$.

12 (a) Suppose $f_n \rightarrow f$ uniformly on a set E in a metric space. Let 'x' be a limit point of E, and suppose that $\underset{t \to x}{Lt} f_n(t) = A_n$ (n =1, 2, 3,....). Then prove that $\{A_n\}$ converges. Also prove that $\underset{t \to x}{Lt} f(t) = \underset{n \to \infty}{Lt} A_n$.

OR

(b) (i) State and prove Weistrass – M test for uniform convergence.
(ii) Prove that C(X) is a complete metric space.

*****