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Code No. 6068 / CBCS

FACULTY OF SCIENCE

M.Sc. I-Semester (CBCS) Examination, December 2016

Subject : Mathematics / Applied Mathematics

Paper - III

Mathematical Methods

Time : 3 hours **Max. Marks : 80**

Note : Answer all questions from Part-A and Part-B. Each question carries 4 marks in Part-A and 12 marks in Part-B.

PART – A (8 x 4 = 32 Marks)

(Short Answer Type)

1 Show that the function $f(t, x) = x^{1/2}$ does not satisfy the Lipschitz condition on the rectangle
$R = \{ (t, x) : |t| \le 1, |x| \le 1\}$.

2 Obtain a partial differential equation by eliminating the arbitrary function f from $z = xy + f(x^2 - y^2)$.

3 Classify the partial differential equation

$$y\frac{\partial^2 u}{\partial x^2} + 2x\frac{\partial^2 u}{\partial x \partial y} + y\frac{\partial^2 u}{\partial y^2} = 0$$

4 Solve $y\frac{\partial u}{\partial x} + x\frac{\partial u}{\partial y} = 0$ by the method of separation of variables.

5 Classify the singular points of the differential equation $(1-x^2)y'' - 2xy' + n(n+1)y = 0$, where n is a constant.

6 Express $f(x) = 2x^3 - 6x^2 + x - 3$ interms of Legendre polynomials.

7 Show that $x J_0'(x) = x J_{n-1}(x) - n J_n(x)$.

8 Show that $H_n'(x) = 2x H_n(x) - H_{n+1}(x)$.

PART – B (4 x 12 = 48 Marks)

(Essay Answer Type)

9 a) i) Find the first three successive approximations for the solution of the initial value problem $x' = e^x,\ x(0) = 0$.

ii) Find the eigen values and eigen functions of $x'' + \lambda x = 0,\ 0 \le t \le \pi,\ x(0) = 0,\ x'(\pi) = 0$.

OR

b) i) Solve $z(x+y)p + z(x-y)q = x^2 + y^2$.

ii) Find the complete integral of $6yz - 6pxy - 3qy^2 + pz = 0$ by Charpit's method.

10 a) Reduce $x^2 \frac{\partial^2 z}{\partial x^2} - y^2 \frac{\partial^2 z}{\partial y^2} = 0$ to canonical form and hence solve it.

OR

b) Solve $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, 0 < x < \ell, t > 0$ subject to the boundary conditions $u(0,t) = 0 = u(\ell, t)$. $u(x,0) = x(\ell - x)$ and $\frac{\partial u}{\partial t}(x,0) = 0$.

11 a) Solve $xy'' + (1-2x)y' + (x-1)y = 0$ in series.

OR

b) i) If $m \neq n$, show that $\int_{-1}^{1} P_m(x)\, P_n(x) dx = 0$.

ii) State and prove Rodrigue's formula for Legendre polynomials.

12 a) Show that $e^{\frac{x}{2}\left(z-\frac{1}{z}\right)} = \sum_{n=-\infty}^{\infty} J_n(x) \cdot z^n$.

OR

b) Prove that $H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n}\left(\bar{e}^{x^2}\right)$.

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