FACULTY OF SCIENCE
M.Sc. I-Semester (CBCS) Examination, December 2016
Subject: Mathematics / Applied Mathematics
Paper - III
Mathematical Methods

Time: 3 hours
Max. Marks: 80

Note: Answer all questions from Part-A and Part-B. Each question carries
4 marks in Part-A and 12 marks in Part-B.

PART – A (8 x 4 = 32 Marks)
(Short Answer Type)

1. Show that the function \( f(t, x) = x^{1/2} \) does not satisfy the Lipschitz condition on the
rectangle
\[ R = \{ (t, x) : |t| \leq 1, |x| \leq 1 \}. \]

2. Obtain a partial differential equation by eliminating the arbitrary function \( f \) from
\[ z = xy + f(x^2 - y^2). \]

3. Classify the partial differential equation
\[ y \frac{\partial^2 u}{\partial x^2} + 2x \frac{\partial^2 u}{\partial x \partial y} + y \frac{\partial^2 u}{\partial y^2} = 0 \]

4. Solve \( y \frac{\partial u}{\partial x} + x \frac{\partial u}{\partial y} = 0 \) by the method of separation of variables.

5. Classify the singular points of the differential equation
\[ (1 - x^2)y'' - 2xy' + n(n + 1)y = 0, \text{ where } n \text{ is a constant.} \]

6. Express \( f(x) = 2x^3 - 6x^2 + x - 3 \) in terms of Legendre polynomials.

7. Show that \( x J_n(x) = x J_{n-1}(x) - n J_n(x) \).

8. Show that \( H_n'(x) = 2x H_n(x) - H_{n-1}(x) \).

PART – B (4 x 12 = 48 Marks)
(Essay Answer Type)

9. a) i) Find the first three successive approximations for the solution of the initial
value problem \( x' = e^x, \ x(0) = 0 \).
ii) Find the eigen values and eigen functions of
\[ x'' + \lambda x = 0, \ 0 \leq t \leq \pi, \ x(0) = 0, \ x'(\pi) = 0. \]

OR

b) i) Solve \( z(x+y)p + z(x-y)q = x^2 + y^2 \).
ii) Find the complete integral of \( 6yz - 6pxy - 3qy^2 + pz = 0 \) by Charpit's method.
10. a) Reduce \( x^2 \frac{\partial^2 z}{\partial x^2} - y^2 \frac{\partial^2 z}{\partial y^2} = 0 \) to canonical form and hence solve it.

OR

b) Solve \( \frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \), \( 0 < x < \ell \), \( t > 0 \) subject to the boundary conditions

\( u(0,t) = 0 = u(\ell, t) \). \( u(x, 0) = x(\ell - x) \) and \( \frac{\partial u}{\partial t}(x, 0) = 0 \).

11. a) Solve \( xy'' + (1 - 2x)y' + (x - 1)y = 0 \) in series.

OR

b) i) If \( m \neq n \), show that \( \int_{-1}^{1} P_m(x) P_n(x) dx = 0 \).

ii) State and prove Rodrigue's formula for Legendre polynomials.

12. a) Show that \( e^{\frac{x}{2}(\frac{z}{z} - 1)} = \sum_{n=-\infty}^{\infty} J_n(x) \cdot z^n \).

b) Prove that \( H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} \left( e^{-x^2} \right) \).