FACULTY OF SCIENCE  
Subject : Mathematics  
Paper – IV : Elementary Number Theory  
Time : 3 Hours  
Max. Marks: 80

Note : Answer all questions from Part–A and Part–B. Each question carries 4 marks in Part–A and 12 marks in Part – B.

PART – A (8 x 4 = 32 Marks)  
(Short Answer Type)

1. Use recursion to find the gcd of 18, 30, 60, 75, 132.
2. If \( f_n \) denotes the nth Fermat number, show that \( 641/ f_5 \).
3. Find the digital roots of square numbers.
4. Solve the linear system 
   \[ x \equiv 1 \pmod{3}, \quad x \equiv 2 \pmod{4}, \quad x \equiv 3 \pmod{5} \] 
   using Chinese Remainder theorem.
5. Find the remainder when \( 24^{1947} \) is divided by 17.
6. Compute \( \phi(8), \phi(81), \phi(15625) \) where \( \phi \) is Euler’s Phi function.
7. Verify that 2 is a primitive root modulo 9.
8. Solve the quadratic congruence \( 3x^2 - 4x + 7 \equiv 0 \pmod{3} \).

PART – B (4 x 12 = 48 Marks)  
(Essay Answer Type)

9. (a) Solve the linear Diophantine equation
   \[ 1076x + 2076y = 3076 \] 
   by Euler’s method.  
   OR
(b) Show that LDE \( ax + by = c \) is solvable if and only if \( d/c \) where \( d = (a, b) \). If \( x_0, y_0 \) is a particular solution of LDE then all its solutions are given by
   \[ x = x_0 + \frac{b}{d}t \] 
   and \[ y = y_0 - \frac{a}{d}t \] 
   where \( t \) is an arbitrary integer.

10. (a) Using Pollard Rho method, factor the integer 3893.  
    OR
(b) (i) Using the method of elimination, solve the linear system
   \[ 2x + 3y \equiv 4 \pmod{13} \]
   \[ 3x + 4y \equiv 5 \pmod{13} \]
   (ii) Prove that digital root of the product of twin primes, other than 3 and 5 is 8.
11 (a) (i) State and prove Euler's theorem.
   (ii) Deduce Fermat's Little theorem from Euler's theorem.

   OR

(b) (i) If \( n \) is a positive integer, show that \( \sum_{d|n} \phi(d) = n \).

(ii) Verify that \( M_{11} \) is a composite number and determine if \( M_{19} \) is a prime where \( M_p \) denotes Mersenne prime.

12 (a) (i) Show that \( \alpha = 3 \) is a primitive root modulo 5 and \( 5^2 \). Also prove that \( \alpha = 5 \) is a primitive root modulo 7 and \( \alpha + p = 5 + 7 = 12 \) is a primitive root modulo \( 7^2 \).

(ii) Solve the congruence \( 3x^2 - 4x + 7 \equiv 0 \pmod{13} \).

   OR

(b) (i) State and prove Law of Quadratic reciprocity.

(ii) Evaluate \( (2 \mid 13) \) using Gauss Lemma.