1212-15-505-002

Code No. 8683

FACULTY OF SCIENCE

M.Sc. II – Semester Examination, May / June 2016

Subject: Mathematics

Paper – V (205)
Discrete Mathematics

Time: 3 Hours **Max.Marks: 80**

Note: Answer all questions from Part-A and Part-B.
Each question carries 4 marks in Part-A and 12 marks in Part-B.

PART – A (8X4 = 32 Marks)
[Short Answer Type]

1 Define a poset and dual of a poset. Prove that the dual of a poset is also poset.

2 Show that a lattice with three or fewer elements is a chain.

3 In any Boolean algebra, show that $a \oplus (a * b) = a$ and $a * (a \oplus b) = a$

4 Define a minterm in n variables and prove that there are 2^n minterms.

5 Prove that there is an even number of vertices of odd degree in any graph.

6 Show that k_n is planar for $1 \le n \le 4$.

7 Show that there is a unique path between every two vertices in a tree

8 Prove that a cut set and any spanning tree must have atleast one edge in common

PART – B (4x12 = 48 Marks)
[Essay Answer Type]

9 a) i) Show that every chain is a distributive lattice

ii) Prove that in a complemented distributive lattice $(L, *, \oplus, ^1, 0, 1)$ for $a, b \in L$
$a \le b \Leftrightarrow a * b' = 0 \Leftrightarrow a' \oplus b = 1 \Leftrightarrow b' \le a'$

OR

b) i) Define the terms sub lattice, lattice homomorphism, complete lattice and complemented lattice.

ii) State and prove De Morgan's law in complemented distributive lattice.

10 a) Prove that the following Boolean expressions are equivalent to one another and obtain their sum-of-products canonical form

i) $(x \oplus y) * (x' \oplus z) * (y \oplus z)$

ii) $(x * z) \oplus (x' * y)$

OR

b) i) Prove that the direct product of two Boolean algebras is a Boolean algebra.

ii) State and prove Stone's representation theorem for finite Boolean algebras.

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11 a) i) State and prove Euler's formula for connected planar graphs.

ii) Prove that $k_{3,3}$ is non-planar.

OR

b) i) If G is a connected planar simple graph with e edges and v vertices where $v \geq 3$, then show that $e \leq 3v - 6$.

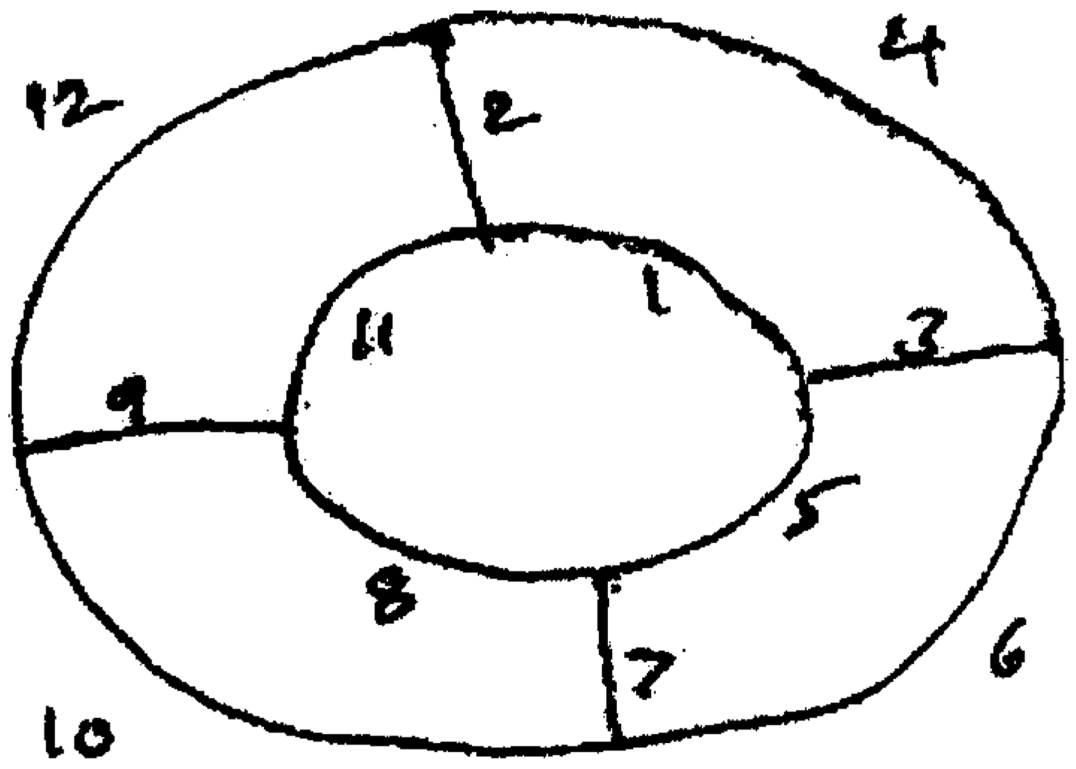
ii) Prove that there is always Hamilton path in a directed complete graph.

12 a) i) Prove that a circuit and complement of any spanning tree must have at least one edge in common.

ii) Prove that a tree with two or more vertices has at least two leaves.

OR

b) i) Determine a minimal spanning tree for the following weighted connected graph.



ii) Prove that a graph with $e = v - 1$ that has no circuit is a tree.

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