FACULTY OF SCIENCE

M.Sc. III – Semester (CBCS) Examination, December 2016
Subject: Mathematics
Paper – IV (B)
Integral Equations

Time: 3 Hours
Max. Marks: 80

Note: Answer all questions from Part-A and Part-B.
Each question carries 4 marks in Part-A and 12 marks in Part-B.

PART – A (8x4 = 32 Marks)
[Short Answer Type]

1 Define Volterra integral equation and Fredholm integral equation.

2 Form an integral equation corresponding to differential equation \( y'' + (1 + x^2)y = \cos x \), \( y(0) = 0, y'(0) = 0 \).

3 Reduce the integral equation of the first kind to second kind \( \int_0^x \sin (x-t) \phi(t) dt = \sin x \).

4 Show that the solution of Abel's problem \( \int_0^x \frac{\phi(t)}{\sqrt{x-t}} dt = c \) is a cycloid.

5 Using Fredholm determinants, find the resolvent Kernel \( k(x,t) = \sin x \cos t, a=0, b=2 \).

6 Define characteristic numbers and eigen function of homogeneous Fredholm integral equation.

7 Show that if Kernel \( K(x,t) \) is symmetric then all its iterated Kernels are symmetric.

8 Construct the Green's function for the BVP \( y'' = 0, y(0) = y(1), y'(0) = y'(1) \).

PART – B (4x12 = 48 Marks)
[Essay Answer Type]

9 a) Solve the integral equation \( \phi(x) = \frac{1}{1+x^2} + \int_0^x \sin (x-t) \phi(t) dt \).

   OR

   b) Explain Picards method of successive approximation. Obtain first four approximations of \( \phi(x) = 2x^2 + 2 - \int_0^x x \phi(t) dt, \phi_0(x) = 2 \).

10 a) Solve the integro-differential equation
\( \phi''(x) + \int_0^x e^{2(x-t)} \phi'(t) dt = e^{2x}, \phi(0) = 0, \phi'(0) = 1 \).

   OR

   b) Derive and solve the Abel's problem.
11 a) Solve the integral equation with degenerate Kernels
\[ \varphi'(x) - \lambda \int_{-\pi}^{\pi} (x \cos t + t^2 \sin x + \cos x \sin t) \varphi(t) \, dt. \]

OR

b) Find the characteristic numbers and eigen function of the integral equation
\[ \varphi(x) = \lambda \int_{0}^{\pi} (\cos^2 x \cos 2t + \cos 3x \cos^3 t) \varphi(t) \, dt. \]

12 a) Explain Schmidt's solution of the non-homogeneous integral equations.

OR

b) Construct the Green's function for the homogeneous BVP
\[ y''(x) = 0, \ y(0) = y'(0) = y(1) = y'(1) = 0. \]