Code No. 6763 / CBCS /N

FACULTY OF SCIENCE

M. Sc. II – Semester Examination, May / June 2017

Subject : Mathematics

Paper – IV: Topology

Time : 3 Hours **Max. Marks: 80**

Note : Answer all questions from Part–A and Part–B. Each question carries 4 marks in Part–A and 12 marks in Part – B.

PART – A (8 x 4 = 32 Marks)
(Short Answer Type)

1 Let $f{:}x \rightarrow y$ be a mapping of one topological space into another. Then show that f is open $\Leftrightarrow$ image of each basic open set is open.

2 Let $x = \{a, b, c\}$. Define a topology T on X and prove that (X, T) is a topological space.

3 Prove that every compact metric space has the Bolzano-Weierstrass property.

4 Show that every sequentially compact metric space is compact.

5 Show that every compact Hausdorff Space is normal.

6 Show that product of any non-empty class of Hausdorff spaces is a Hausdorff space.

7 Show that components of a totally disconnected space are its points.

8 Give an example of topological space which is connected but not locally connected.

PART – B (4 x 12 = 48 Marks)
(Essay Answer Type)

9 (a) Let X be any non-empty set and let S be an arbitrary class of subsets of X. Then prove that S can serve as an open base for a topology on X, in the sense that the class of all unions of finite intersections of sets in S is a topology on X.

OR

(b) (i) State and prove Lindelof's theorem.
(ii) Show that every seperable metric space is second countable.

10 (a) State and prove Ascolis theorem.

OR

(b) (i) State and prove Lebesgue's covering Lemma.
(ii) Show that every sequentially compact metric space in totally bounded.

11 (a) (a) State and prove Urysohn's lemma.

OR

(b) (i) Prove that in a T_2 space any point and disjoint compact subspace can be separated by open sets, in the sense they have disjoint neighbourhoods.
ii) State and prove Tietze's extension theorem.

12 (a) (i) Show that a subspace of real line R is connected $\Leftrightarrow$ it is an interval. In particular prove that R is connected.
(ii) Prove that continuous image of a connected space is connected.

OR

(b) (i) Show that components of a totally disconnected space are its points.
(ii) Let X be a compact T_2-sapce. Then show that X is totally disconnected $\Leftrightarrow$ it has an open base whose sets are also closed.

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