Code No. 7072

FACULTY OF SCIENCE

M.Sc. IV-Semester Examination, May / June 2017

Subject : Mathematics

Paper - II

General Measure Theory

Time : 3 hours **Max. Marks : 80**

Note : Answer all questions from Part-A and Part-B. Each question carries 4 marks in Part-A and 12 marks in Part-B.

PART – A (8 x 4 = 32 Marks)

(Short Answer Type)

1 Define a finite measure space. If $A \in \mathcal{B}$, $B \in \mathcal{B}$ and $A \subset B$ then show that $\mu A \leq \mu B$.

2 Let f be an extended real valued function defined on x. Then prove that

$\{ x : f(x) \leq \alpha \} \in \mathcal{B}$ for each α if and only if

$\{ x : f(x) > \alpha \} \in \mathcal{B}$ for each α.

3 Define a signed measure on a measurable space (x, β). Prove that union of countable collection of positive sets is positive.

4 State and prove Hahn decomposition theorem.

5 If $A \in \mathcal{A}$ and if $\{A_i\}$ is any sequence of sets in $\mathcal{A}$ such that $A \subset \bigcup_{i \geq 1}^{\infty} A_i$. Then show that

$\mu A \leq \sum_{i \geq 1}^{\infty} \mu A_i$.

6 Show that the set function μ^* is an outer measure.

7 Let B be a μ^* - measurable set with $\mu^* B < \infty$. Then prove that $\mu_* B = \mu^* B$.

8 Define an inner measure μ_* induced by a measure μ on an algebra $\mathcal{A}$ of subsets of x. Also prove that if $E \in \mathcal{A}$ then $\mu_* E = \mu E$.

PART – B (4 x 12 = 48 Marks)

(Essay Answer Type)

9 a) i) If $E_i \in \mathcal{B}$, $\mu E_1 < \infty$ and $E_i \supset E_{i+1}$, then show that $\mu\left(\bigcap_{i=1}^{\infty} E_i\right) = \underset{n \to \infty}{Lt}\ \mu E_n$.

ii) State and prove Lebesque convergence theorem.

OR

b) i) State and prove Fatou's Lemma.

ii) If f and g are non-negative measurable functions and a and b non-negative constants, then show that $\int af + bg = a\int f + b\int g$. Further show that $\int f \geq 0$ holds with equality only if f = 0 a.e.

10 a) Let E be a measurable set such that $0 < \nu E < \infty$. Then prove that there is a positive set A contained in E with $\nu A > 0$.

OR

b) State and prove Radon-Nikodym theorem.

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11 a) Show that the class $\mathscr{B}$ of μ^* - measurable. Sets is a σ - algebra. If $\bar{\mu}$ is μ^* restricted to $\mathscr{B}$, then prove that $\bar{\mu}$ is a complete measure on $\mathscr{B}$.

OR

b) State and prove Fubini theorem.

12 a) Let E and F be disjoint sets. Then show that $\mu_* E + \mu_* F \le \mu_* (E \cup F) \le \mu_* E + \mu^* F \le \mu^* (E \cup F) \le \mu^* E + \mu^* F$.

OR

b) Let $\{A_i\}$ be a disjoint sequence of sets in $\mathscr{A}$.

Then show that $\mu_*\left(E \cap \bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} \mu_*(E \cap A_i)$.

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