Note: Answer all questions from Part-A and Part-B. Each question carries 4 marks in Part-A and 12 marks in Part-B.

PART - A (8 x 4 = 32 Marks)
(Short Answer Type)

1. Define a finite measure space. If \( A \in \mathcal{B}, B \in \mathcal{B} \) and \( A \subset B \) then show that \( \mu A \leq \mu B \).

2. Let \( f \) be an extended real valued function defined on \( x \). Then prove that 
   \[ \{ x : f(x) \leq \alpha \} \in \mathcal{B} \text{ for each } \alpha \text{ if and only if} \]
   \[ \{ x : f(x) > \alpha \} \in \mathcal{B} \text{ for each } \alpha \]

3. Define a signed measure on a measurable space \((X, \mathcal{B})\). Prove that union of countable collection of positive sets is positive.

4. State and prove Hahn decomposition theorem.

5. If \( A \in \mathcal{A} \) and if \( \{A_n\} \) is any sequence of sets in \( \mathcal{A} \) such that \( A = \bigcup_{n=1}^{\infty} A_n \). Then show that 
   \[ \mu A \leq \sum_{n=1}^{\infty} \mu A_n \, \text{.} \]

6. Show that the set function \( \mu \) is an outer measure.

7. Let \( B \) be a \( \mu \)-measurable set with \( \mu B < \infty \). Then prove that \( \mu B = \mu^* B \).

8. Define an inner measure \( \mu^* \) induced by a measure \( \mu \) on an algebra \( \mathcal{A} \) of subsets of \( x \).
   Also prove that if \( E \in \mathcal{A} \) then \( \mu^* E = \mu E \).

PART - B (4 x 12 = 48 Marks)
(Essay Answer Type)

9. a) i) If \( E_1 \subset E_2 \), \( \mu E_1 < \infty \) and \( E_1 \supset E_{n+1} \), then show that 
      \[ \mu \left( \bigcap_{t=1}^{\infty} E_t \right) = \lim_{n \to \infty} \mu E_n \, . \]
      ii) State and prove Lebesgue convergence theorem.

      OR

   b) i) State and prove Fatou's Lemma.
      ii) If \( f \) and \( g \) are non-negative measurable functions and \( a \) and \( b \) non-negative constants, then show that 
          \[ \int [af + bg] = a \int f + b \int g \, . \]
          Further show that \( \int f \geq 0 \) holds with equality only if \( f = 0 \) a.e.

10. a) Let \( E \) be a measurable set such that \( 0 < \mu E < \infty \). Then prove that there is a positive set \( A \) contained in \( E \) with \( \nu A > 0 \).

      OR

   b) State and prove Radon-Nikodym theorem.
11 a) Show that the class $\mathcal{B}$ of $\mu^*$-measurable sets is a $\sigma$-algebra. If $\mu^*$ is restricted to $\mathcal{B}$, then prove that $\mu^*$ is a complete measure on $\mathcal{B}$.

OR

b) State and prove Fubini theorem.

12 a) Let $E$ and $F$ be disjoint sets. Then show that $\mu_\ast E + \mu_\ast F \leq \mu_\ast (E \cup F) \leq \mu_\ast E + \mu_\ast F$.

OR

b) Let $\{A_i\}$ be a disjoint sequence of sets in $\mathcal{A}$. Then show that $\mu_\ast \left( \bigcup_{i=1}^{\infty} A_i \right) \leq \sum_{i=1}^{\infty} \mu_\ast (E \cap A_i)$.