FACULTY OF SCIENCE
M.Sc. IV-Semester Examination, May / June 2017
Subject: Mathematics/Applied Maths
Paper - V (a)
Calculus of Variations

Time: 3 hours
Max. Marks: 80

Note: Answer all questions from Part-A and Part-B. Each question carries 4 marks in Part-A and 12 marks in Part-B.

PART - A (8 x 4 = 32 Marks)
(Short Answer Type)

1. Define a linear functional and variational functional. Give one example each.
2. On what curves can the functional
   \[ V[y(x)] = \int_{0}^{\pi/2} \left( y'^2 - y^2 \right) dx; \quad y(0) = 0, \ y(\pi/2) = 11 \]
   be extremized.
4. Find the extremals of the functional
   \[ V[y(x), z(x)] = \int_{0}^{\pi/2} \left( y'^2 - z^2 + 2y^2z \right) dx. \]
5. Find the extremals of the functional
   \[ V[y(x), z(x)] = \int_{x_0}^{x_1} \left( 16y^2 - x^2 + x^2 \right) dx. \]
6. Find the Euler-Ostrogradsky equation for the functional
   \[ S[z(x,y)] = \int \sqrt{1 + \left( \frac{dz}{dx} \right)^2 + \left( \frac{dz}{dy} \right)^2} \ dx \ dy. \]
7. Derive the equations of motion of a projectile in space using Hamilton’s equation.
8. Derive the differential equation of motion of simple pendulum using Lagrange’s equation.

PART - B (4 x 12 = 48 Marks)
(Essay Answer Type)

9. a) i) State and prove the fundamental Lemma of calculus of variations.
    ii) Prove that the shortest distance between two points in a place is a straight line.

    OR

b) Derive Euler’s equation for the functionals of the form
   \[ V[y(x)] = \int_{x_0}^{x_1} F(x,y,y') dx, \quad y(x_0) = y_0, \ y(x_1) = y_1. \]
10 a) Define minimum surface of revolution problem and show that it is a family of catenaries.

OR

b) Find the extremals of the functional
\[ V[y(x), z(x)] = \int_0^{\pi/2} \left[ y'^2 + z'^2 + 2yz \right] dx, \quad y(0) = 0, \quad y(\pi/2) = 1, \quad z(0) = 1, \quad z(\pi/2) = -1. \]

11 a) State and derive isometric problem.

OR

b) Derive the Euler-Ostrogradsky equation for the functional
\[ V[z(x,y)] = \int_0^l F(x, y, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}) \, dx \, dy. \]

12 a) Derive the differential equation of the free vibrations of a string using the variational principle.

OR

b) Derive the Euler-Poisson equation.