Code No. 2060 / CORE

FACULTY OF SCIENCE

M.Sc. I-Semester Examinations, January 2018

Subject : Maths/ Applied Maths/ Maths with Computer Science

Paper – I

Algebra

Time: 3 hours **Max. Marks: 80**

PART-A (8 x 4 = 32 Marks)

(Short Answer Type)

1 If G is a group and H<G of finite index n then prove that there is a homomorphism $\Phi = G \to S_n$ such that $\text{Ker}\Phi = \bigcap_{x \in G} xHx^{-1}$.

2 Define solvable group and nilpotent group

3 Find the non isomorphic abelian groups of order 360.

4 Prove that a finite group G is a p – group if and only if its order is a power of p.

5 Show that an ideal M in the ring of integers Z is a maximal ideal if and only if M= (p) where p is some prime

6 In a nonzero commutative ring with unity prove that an ideal M is maximal if and only if $\frac{R}{M}$ is a field.

7 Prove that every Euclidean domain is a PID.

8 Prove the ring of Gaussian integers $R = \{m + n\sqrt{-1} / m, n \in Z\}$ is a Euclidean domain.

PART-B (4 x 12 = 48 Marks)

(Essay Answer Type)

9 a) State and prove Burnside theorem.

OR

b) If G is a nilpotent group then prove that every sub group of G and every homomorphic image of a are nilpotent

10 a) Let A be a finite abelian group. Then prove that there exists a unique list of integers $m_1, m_2, ... m_k$ (all>1) such that $|A| = m_1 m_2 ... m_k$ where $m_1 | m_2 | ... m_k$ and $A = c_1 \oplus c_2 \oplus ... \oplus c_k$ where $c_1, c_2, ..., c_k$ are cyclic subgroups of A of order $m_1, m_2, ... m_k$ respectively.

OR

b) State and prove second and third sylow theorems.

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11 a) For any ring R and ideal $A \neq R$ prove that the following are equivalent :

i) A is maximal.

ii) The quotient ring $\frac{R}{A}$ has no nontrivial ideals.

iii) For any element $x \in R, X \notin A, A + (x) = R$

OR

b) If R is a non-zero ring with unity and I is an ideal in R such that $I \neq R$ then prove that there exists a maximal ideal M of R such that $I \subseteq M$.

12. a) Prove that Every PID is a UFD but a UFD need not be a PID.

OR

b) If R is a unique factorization domain then prove that the polynomial ring R(x) over R is also a unique factorization domain.

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