Code No. 2062 / CORE

FACULTY OF SCIENCE

M.Sc. I-Semester Examinations, January 2018

Subject : Maths / Applied Maths

Paper – III

Mathematical Methods

Time: 3 hours **Max. Marks: 80**

Note : Answer all questions from Part-A and Part-B. Each question carries 4 marks in Part-A and 12 marks in Part-B.

PART-A (8 x 4 = 32 Marks)
(Short Answer Type)

1 Apply Picard's method for the Initial value problem $\frac{dy}{dx} = 1 + xy$, $y(0) = 2$ and find the first three successive approximations.

2 Form a partial differential equation by eliminating arbitrary function f and F from space $y = f(x - at) + F(x + at)$.

3 Classify and find the characteristics of $4r + 5s + t + p + q - 2 = 0$.

4 Solve one dimensional wave equation by using method of separation of variables.

5 If $P_n(x) = \sum_{r=0}^{\left[\frac{n}{2}\right]} \frac{(-1)^r \,(2n-2r)!\, x^{n-2r}}{2^r\, r!\,(n-r)!\,(n-2r)!}$

6 Define ordinary and singular points of the differential equation $y'' + P(x)y' + Q(x)y = 0$ and classify the singular points.

7 If n is an integer then prove that $J_{-n}(x) = (-1)^n J_n(x)$.

8 If $H_n(x)$ is an Hermite polynomial of order n then prove that $H_{2n}(0) = (-1)^n \frac{(2n)!}{n!}$.

PART-B (4 x 12 = 48 Marks)
(Essay Answer Type)

9 a) Prove that all eigen values of strum Liouville problem are real.

OR

b) Prove that the general solution of Lagrange equation $Pp + Qq + R$ is $\Phi(u, v) = 0$ where Φ an arbitrary function and $u(x, y, z) = c_1, v(x, y, z) = c_2$ are two independent solutions of $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$, where c_1, c_2 are arbitrary constants and at least one of u, v must contain z.

10 a) Reduce the equation $yr + (x + y)s + xt = 0$ to canonical form and find its general solution.

OR

b) An insulated rod of length l has its ends A and B maintained at 0°C and 100°C respectively until steady state condition prevails. If B is suddenly cooled to 0°C, and maintained at 0°C, then find the temperature at a distance x from A at time t.

11 a) Prove that $(1-2xz+z^2)^{\frac{-1}{2}} = \sum_{n=0}^{\infty} z^n P_n(x)$, where $|x| \le 1, |z| < 1$

OR

b) Solve in series the differential equation $xy'' + y' + xy = 0$

12 a) Prove that $\int_0^a x J_n(\lambda_i x) J_n(\lambda_j x)dx = \frac{a^2}{2} J_{n+1}(\lambda_i a)\delta_{ij}$ where $\delta_{ij} = 0$ if $i \neq j$ and $\delta_{ij} = 1$ if $i = j$, whenever λ_i and λ_j are the roots of the equation $J_n(\lambda a) = 0$.

OR

b) Prove that $H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2}$

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