FACULTY OF SCIENCE
M.Sc. II – Semester Examination, May / June 2018
Subject: Mathematics
Paper – IV : Topology
Time : 3 Hours
Max. Marks: 80

Note: Answer all questions from Part–A and Part–B. Each question carries 4 marks in Part–A and 12 marks in Part – B.

PART – A (8 x 4 = 32 Marks)
(Short Answer Type)

1. Let X be an infinite set. Show that the empty set $\emptyset$ together with all subsets of X whose complements are finite is a topology on X.

2. Let X be a topological space and A an arbitrary subset of X. Then prove that $\tilde{A} = \{x :$ each neighbourhood of $x$ intersects $A\}$

3. Show that any continuous image of a compact space is compact.

4. Prove that totally bounded metric space is bounded.

5. Show that a closed subspace of a normal space is normal.

6. Show that a topological space is a $T_1$ – Space if and only if each point is a closed set.

7. Define component of a topological space $x$, and prove that each point in $X$ is contained in exactly one component of $X$.

8. Let X be a topological space. If $\{A_i\}$ is a non-empty class of connected subspaces of X such that $\bigcap A_i$ is non-empty, then prove that $A = \bigcup A_i$ is also a connected subspace of X.

PART – B (4 x 12 = 48 Marks)
(Essay Answer Type)

9. (a) Let X be a non-empty set and let there be given a class of subsets of X which is closed under the formation of arbitrary intersections and finite unions. Then show that the class of all complements of these sets is a topology on X whose closed sets are precisely those initially given.

OR

(b) Let X be any non-empty set, and let S be an arbitrary class of subsets of X. Then prove that S can serve as an open subbase for a topology on X, in the sense that the class of all unions of finite intersections of sets in S is a topology.

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10 (a) State and prove Ascoli’s theorem.
   OR
   (b) (i) Show that a metric space is sequentially compact ⇔ it has the Bolzano Weierstrass property.
   (ii) Show that a closed subspace of a complete metric space is compact ⇔ it is totally bounded.

11 (a) State and prove Urysohn’s imbedding theorem.
   OR
   (b) State and prove Tietze’s extension theorem.

12 (a) Show that a subspace of real line $\mathbb{R}$ is connected if and only if it is an interval. Hence prove that $\mathbb{R}$ is connected.
   OR
   (b) (i) Let $X$ be a compact Hausdorff space. Then prove that $X$ is totally disconnected ⇔ it has an open base whose sets are also closed.
   (ii) Let $X$ be a locally connected space. Prove that if $Y$ is an open subspace of $X$, then each component of $Y$ is open in $X$. In particular, each component of $X$ is open.

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