1212-16-505-001

Code No. 2449 / Core

FACULTY OF SCIENCE

M.Sc. III – Semester Examination, January 2018

Subject: Mathematics

Paper – II : Functional Analysis

Time: 3 Hours **Max.Marks: 80**

Note: Answer all questions from Part-A and Part-B.
Each question carries 4 marks in Part-A and 12 marks in Part-B.

PART – A (8x4 = 32 Marks)
[Short Answer Type]

1. In a normed linear space prove that an open ball is a convex set.
2. Let C[0,1] be the space of all real valued continuous functions defined on [0,1] then prove that C[0,1] is a normed linear space with the defined norm
   $||x|| = \max_{t \in [0,1]} |x(t)|, \ x \in c[0,1].$
3. State and prove Cauchy-Bunyakovsky Schwartz inequality.
4. Prove that the space ℓ_p with $p \neq 2$ is not a Hilbert space.
5. If $\{A_n\}$ converge uniformly to A then prove that $\{A_n\}$ is point wise convergent.
6. Let A be an operator on a normed linear space E_x then prove that
   i) $||Ax|| \leq ||A|| \, ||x|| \ \forall x \in E_x$.
   ii) For every $\epsilon > 0$, there is an element x_ϵ such that $||Ax_\epsilon|| > (||A|| - \epsilon) \, ||x_\epsilon||$.
7. Define
   i) Graph of an operator
   ii) Closed linear operator
   iii) Projection operator
8. Define self adjoint operator. Also prove that if A, B are self adjoint operators then A+B is also self adjoint.

PART – B (4x12 = 48 Marks)
[Essay Answer Type]

9 a) Prove that a subspace X of a Banach space E is complete if and only if the set X is closed in E.

OR

b) Let Y and Z be subspaces of a normed linear space X and suppose that Y is closed and is a proper subset Z. Then for every real number $\theta \in (0,1)$ there is a $z \in Z$ such that $||z||=1$, $||z-y|| \geq \theta \ \forall y \in Y$.

10 a) Let H be a Hilbert space and L is closed, convex set in H and $x \in H - L$ then there is a unique $y_o \in L$ such that $||x - y_o|| = \inf_{y \in L} \{||x-y||\}$.

OR

b) Prove that a subspace L of a Hilbert space H is closed if and only if $L = L^{\perp\perp}$.

11 a) State and approve generalized Hahn-Banach theorem.

OR

b) State and prove uniform boundedness principle.

12 a) State and prove open mapping theorem.

OR

b) Prove that every self adjoint operator p satisfying $p^2=p$ is an orthogonal projection on some subspace L of the Hilbert space H.

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