FACULTY OF SCIENCE
M.Sc. III – Semester Examination, January 2018
Subject: Mathematics
Paper – III (A)
Discrete Mathematics

Time: 3 Hours Max. Marks: 80

Note: Answer all questions from Part-A and Part-B.
Each question carries 4 marks in Part-A and 12 marks in Part-B.

PART – A (8x4 = 32 Marks)
[Short Answer Type]

1. Let L be a lattice. Then for every a and b in L,
   a) \( a \lor b = b \) if and only if \( a \leq b \),
   b) \( a \land b = a \) if and only if \( a \leq b \),
   c) \( a \land b = a \) if and only if \( a \lor b = b \).

2. Show that the lattices pictured in the following figure are non-distributive.

   ![Lattices Diagram]

3. Show that if \( n \) is a positive integer and \( p^2/n \), where \( p \) is a prime number, then prove that
   \( D_n \) is not a Boolean algebra.

4. Show that in a Boolean algebra, for any \( a \) and \( b \),
   \( (a \land b) \lor (a \land b') = a \).

5. If a graph \( G \) has more than two vertices of odd degree, then prove that there can be no
   Euler path in \( G \).

6. Let the number of edges of \( G \) be \( m \). Then prove that \( G \) has a Hamiltonian circuit if
   \( m \geq \frac{1}{2} (n^2 - 3n + 6) \) (here \( n \) is the number of vertices).

7. Let \( (T, v_0) \) be a rooted tree on a set \( A \). Then prove that
   a) \( T \) is irreflexive
   b) \( T \) is asymmetric
   c) If \( (a, b) \in T \) and \( (b, c) \in T \), then \( (a, c) \notin T \), for all \( a, b, \) and \( c \) in \( A \).

8. Prove that a tree with \( n \) vertices has \( n - 1 \) edges.
PART – B (4x12 = 48 Marks)
[Essay Answer Type]

9 a) If \( s_1 = \{x_1, x_2, \ldots, x_n\} \) and \( s_2 = \{y_1, y_2, \ldots, y_n\} \) are any two finite sets with \( n \) elements, then prove that the lattices \( (p(s_1), \subseteq) \) and \( (p(s_2), \subseteq) \) are isomorphic.

OR

b) Let \( L \) be a lattice. Then \( L \) holds the following:
   1) Idempotent properties
   2) Commutative properties
   3) Associative properties
   4) Absorption properties.

10 a) Show that in a Boolean algebra for any \( a, b, \) and \( c \):
   \[
   (a \land b \land c) \lor (b \land c) = b \land c.
   \]

OR

b) Let \( n = p_1 p_2 \ldots p_k \) where the \( p_i \) are distinct primes. Then prove that \( D_n \) is a Boolean algebra.

11 a) What is the total number of edges in \( K_n \), the complete graph on \( n \) vertices? Justify your answer.

OR

b) Draw the complete graph on seven vertices.

12 a) Let \((T, v_0)\) be a rooted tree. Then prove the following:
   i) There are no cycles in \( T \)
   ii) \( v_0 \) is the only root of \( T \)
   iii) Each vertex in \( T \), other than \( v_0 \), has in-degree one, and \( v_0 \) has in-degree zero.

OR

b) If \((T, v_0)\) is a rooted tree and \( v \in T \), then prove that \( T(v) \) is also a rooted tree with root \( v \).