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Code No. 3381/CORE

FACULTY OF SCIENCE
M.Sc. IV – Semester Examination, May / June 2018

Subject: Mathematics / Applied Mathematics

Paper – III (A): Integral Equations and Calculus of Variations

Time : 3 Hours **Max. Marks: 80**

Note : Answer all questions from Part–A and Part–B. Each question carries 4 marks in Part–A and 12 marks in Part – B.

PART – A (8 x 4 = 32 Marks)
(Short Answer Type)

1 Find the resolvent Kernel of the Volterra integral equation with Kernel $K(x, t) = e^{x-t}$.
2 Show that $\beta(p, q) = \beta(p+1, q)+\beta(p, q, + 1)$.
3 Show that the homogeneous Integral equation

$\phi(x) = \lambda\int_0^1 (3x-2)t\varphi(t)dt = 0$ has no characteristic numbers and eigen functions.

4 Define Green's function and find Green's function for the boundary – value problem $y'' - y = 0$; $y(0)=y(1)=0$.

5 On what curves can the functional $v[y(x)] = \int_0^{\pi/2} [(y')^2 - y^2]\, dx$; $y(0) = y(\pi/2) = 1$,

be extremized.
6 Find the extremal of the functional

$$v[y(x)] = \int_{x_0}^{x_1} (zxy + y'''^2)\, dx$$

7 Write the ostrogradsky equation for the functional

$$v = \iint_D \left[\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 2zf(x, y)\right]dxdy$$

8 State and prove Hamilton's principle.

PART – B (4 x 12 = 48 Marks)
(Essay Answer Type)

9 (a) With aid of resolvent Kernel, find the solution of the integral equation

$$\varphi(x) = x3^x - \int_0^x 3^{x-t}\varphi(t)dt$$

OR

(b) Solve the integro-differential equation.

$$\varphi''(x) + \int_0^x e^{2(x-t)}\varphi'(t)dt = e^{2x}\ \ \varphi(0) = 0, \varphi'(0) = 1$$

10 (a) Solve the integral equation

$$\varphi(x) - \lambda \int_{-\pi}^{\pi} (x \cos t + t^2 \sin x + \cos x \sin t)\varphi(t)dt = x$$

OR

(b) Using the Green's function, solve to boundary value problem
$y'' = y = x^2$; $y(0) = y(\pi/2)=0$

11 (a) Show that the externals of the functional $t[y(x)] = \int_{x_0}^{x_1} \frac{\sqrt{1+y^2}}{x} dx$ are family of circles

OR

(b) Define minimum-surface of revolution problem and solve it.

12 (a) Derive the Euler-Poisson equation for the functional dependent on higher-order-derivatives.

OR

(b) Derive the differential equation of the free vibration of a string using the varitional principle.

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