FACULTY OF SCIENCE
M. Sc. I – Semester (CBCS / Non-CBCS) Examination, December 2013
Subject: Physics and Applied Electronics

Paper – I: Mathematical Physics

Time: 3 Hours  Max. Marks: 80

Note: Answer all questions from Part-A and Part-B. Each question carries 4 marks in Part-A and 12 marks in Part - B.

PART - A (8 x 4 = 32 Marks)

(Short Answer Type)

1. Prove the recurrence relation
   \[ n P_n = (2n-1) x P_{n-1} - (n-1) P_{n-2} \]
   for Legendre's polynomials.

2. Define gamma function. Show that
   \[ \Gamma \left( \frac{1}{2} \right) = \sqrt{\pi} \]

3. Prove the recurrence relation
   \[ H_n(x) = 2n H_{n-1}(x) \]
   for Hermite polynomials.

4. Obtain Rodrigues formula for Hermite polynomial.

5. Show that the Fourier transform of \( f(at) = \frac{1}{a} g \left( \frac{w}{a} \right) \)
   Where \( g(w) \) is the Fourier transform of \( f(t) \).

6. Find the Laplace transform of \( t \sin at \).

7. Show that every square matrix can be expressed as the sum of a Hermitian and
   Skew-Hermitian matrix.

8. Explain the inner product of two tensors.

PART - B (4 x 12 = 48 Marks)

(Essay Answer Type)

5. (a) Obtain the power series solution of Legendre's differential equation and show that
   \[ P_n(x) = \frac{1}{2} (5x^2 - 3x) \]

   (b) Obtain the polynomial solution of Bessel's differential equation and prove that
   \( J_n(x) = n J_{n+1}(x) \).

10. (a) Set up the wave equation for the vibrations of a rectangular membrane and find its
     solution.

     OR

     (b) Show that the Hermite polynomials are generated by the function \( e^{2x^2} \) and hence
         prove that
         \[ H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} (e^{-x^2}) \]

11. (a) What is Laplace and inverse Laplace transform of a function \( f(t) \). State and prove the
     convolution theorem for Laplace transform.

     OR

     (b) Find the Fourier transform of
         \( (i) f(x) = Ne^{-\alpha x^2} \) \( N \) and \( \alpha \) are constants \( (ii) e^{3t} \)

12. (a) What are Christoffel's symbols of First and Second kind? Establish a relation between
     them.

     OR

     (b) What is the characteristic equation of a matrix? Find the eigen values and eigen vectors
     of the matrix.

     \[
     \begin{bmatrix}
     1 & 1 & 1 \\
     1 & 2 & 3 \\
     2 & 6 & 4
     \end{bmatrix}
     \]

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